Energy Efficient Information Dissemination in Ad Hoc Networks Utilizing Bi-orthogonal Signaling

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Abstract *In this paper, we leverage bi-orthogonal signaling by using a time slot dependent identification scheme that enables energy efficient information dissemination, independent of the network topology. The approach, although motivated by cooperative modulation schemes that require information to be dispersed to other nodes within a wireless ad hoc network, can be applied directly to wired networks. As a measure of energy efficiency in information dissemination, we consider the number of transmissions required for broadcast. For the grid topology, we give a tighter bound than [1] without using our proposed time slot dependent identification scheme, and then focus on optimizing an information dissemination scheme for the grid topology.*

Keywords: cooperative communications, energy efficiency, sensor networks.

I. INTRODUCTION

In a *cooperative sensing network* [2], the primary objective is to extract the aggregated information from the distributed sensors. Moreover, there is a strong desire to utilize techniques that will extend the battery life of the network.

In this paper, we consider cooperative techniques for energy efficient communication. The wireless ad hoc network contains a set of nodes distributed in a plane where each node is assumed to have an omnidirectional antenna. We consider the commonly used power-attenuation model such that the power needed to sustain a link uv is $\|uv\|^{\beta}$, $\|uv\|$ is the Euclidean distance between u and v, and $2 \leq \beta \leq 4$ models the attenuation due to various environmental factors[3], [4], [5].

To measure energy efficiency, Li et al. [6] defined the *power stretch factor*[*]. The power stretch factor is used to measure energy efficiency of a subgraph on a per path basis. In this paper, we want to minimize the total number of transmissions needed for broadcast, which concerns the entire network (global optimization) rather than a specific path and so we examine a methodology for minimizing the total number of transmissions required to disseminate a fixed set of information[†], motivated by the goal of reducing the total energy utilized per bit in terms of a transmission.

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[*]This is similar to the length stretch factor, also called the dilation ratio or spanning ratio.

[†]We assume these techniques are independent of data fusion methods which in general may be employed simultaneously with the work in this paper.

II. PRELIMINARIES

We assume that an external resource such as a satellite or some other source capable of generating sync tones is used to synchronize the time-slots of the nodes before data communication begins. We also assume there is an existing schedule, so that each node uses a pre-assigned time-slot.

Let V be the set of N nodes in a plane $\mathbb{R}^2$, representing N nodes in a network. Each node contains D bits of information that it must share with all the others, resulting in all N nodes containing an identical set of ND bits. Moreover, let E be the set of valid edges for the defined topology, and G be a graph where two nodes are connected in G if and only if the nodes can hear each other when using maximum transmission power during separate transmissions.

A. Metric

Multicast gossip is all-to-all communication [1]. To minimize the number of transmissions in multicast gossip, we cover all the nodes of the network with the minimum number of disks, where each disk represent a transmission from the node located at the center of the disk. Note that, this problem differs from the minimum disk cover problem because our cover must follow admissible waves of transmissions, where each wave originates from a source node and propagates toward all other nodes.

Definition 1 (wave-cover number ϖ_G) Let τ_v be the number of transmissions made by node v during the multicast gossip. Then the wave-cover number is $\varpi_G = \sum_{v \in V} \tau_v$.

In the remainder of this paper, we shall consider the constrained case where $\|uv\|$ is fixed.

B. A brief review of NSOC cooperative communication

The ND bits of information to be shared are partitioned into groups of $\lfloor \log_2 N \rfloor + 1$ bits, resulting in $\lceil \frac{ND}{\lfloor \log_2 N \rfloor + 1} \rceil$ groups of bits; zero padding is used for partial groups. Since all N nodes have the identical ND time ordered bits in groups, the first $\lfloor \log_2 N \rfloor$ bits in each group determine which one of the N nodes will transmit the last bit in that group. This process is repeated until all ND bits have been forwarded to a satellite via FDMA or some other bi-orthogonal signaling scheme.

Consider the generalized form of the optimal receiver structure for AWGN where the receiver simultaneously correlates based on time slots viewed as depicted in Figure 1, originally presented in [7] and refined for NSOC in [1].
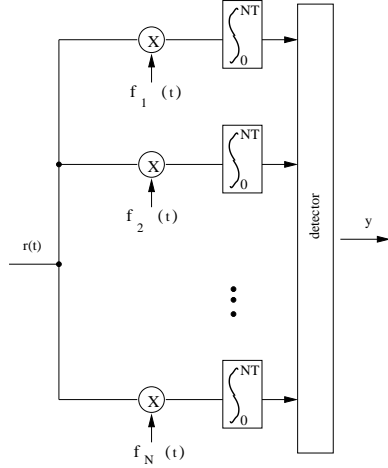

Fig. 1. Optimal bi-orthogonal signaling receiver where $r(t)$ is the received signal. For $i = 1, 2, \ldots, N$, each correlation function $f_i(t)$ is such that transmitted signal $s(t)$ multiplied by the $f_i(t)$ is $\frac{1}{NT}$ for interval $(i-1)T$ to iT if $s(t) = f_i(t)$, else $s(t)f_i(t) = 0$. The matched filters integrate over period NT, and y is the hard decision output of the receiver.

Independent of the modulation scheme, we can employ a higher OSI layer form of bi-orthogonality assuming bipolar signaling[‡] as a minimum criteria. We extend this idea of uniqueness of a node (or time slot) as a form of orthogonality to the process of information dissemination. This idea can also be viewed as pulse position modulation where the pulse contains a bit of information, or as source coding, or as an extension of the bi-orthogonality signaling on the hard decision data.

III. TIME SLOT IDENTIFICATION FOR INFORMATION DISSEMINATION

Time Slot Identification for Information Dissemination (TSIID) is an energy efficient form of information dissemination among a set of nodes. TSIID could be used for energy efficient information dissemination among a set of wireless nodes prior to forwarding data to a satellite. We assume the nodes are assigned unique identifier numbers from 0 to $N-1$. In the following, we use n_i to denote a node whose identifier is i. The framing structure contains three levels: the frames, the block frames and the super frames. Figure 2 depicts the time ordering and framing for the N nodes.

A. Frames

The lowest level of the framing structure of TSIID contains $\hat{N}$ slots per node. The time slot where a bit is placed

[‡]Note that our scheme requires bipolar signaling to generate a "valid" flag to resolve when no information is transmitted in a time slot.
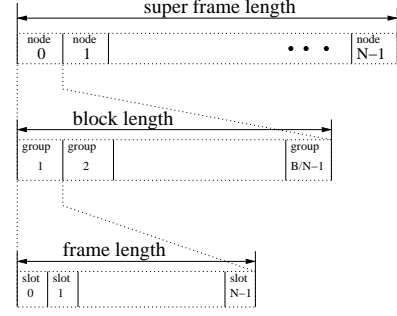

Fig. 2. Framing structure of Time Slot Identification for Information Dissemination

shall resolve $\lfloor \log_2 \hat{N} \rfloor$ bits from a single node, where the actual bit transmitted shall be the $(\lfloor \log_2 \hat{N} \rfloor + 1)^{th}$ bit of the D bits from a node, and $\hat{N}$ is the number of unique time-slots associated with the network. We call the set of $(\lfloor \log_2 \hat{N} \rfloor + 1)$ bits a *group* where a single transmitted bit in a specific time slot provides the equivalent information of the entire group of bits. Thus, the number of actual bits transmitted is less than the actual amount of raw bits to be disseminated among the other $N-1$ nodes. Without loss of generality, we assume $\hat{N} = N$.

Each node is assigned a time slot within a frame. Each node has D bits of information to be shared with "some set of nodes"; in the worst case, the set contains all other nodes. As an upper bound, we assume that a bit shall propagate to all other $N-1$ nodes. Using this upper bound, the total number of transmissions required to disseminate ND bits will hold for all other cooperative modulation schemes in [1], including NSOC.

B. Block Frames

Each node partitions the D bits into groups of $\lfloor \log_2 N \rfloor + 1$ bits. Thus, each node contains $\lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \rceil$ groups, where zero padding resolves any partial groups. Since in a multihop network, a node may need to forward information from other nodes, and there are N nodes in the network, we use $N \lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \rceil$ time slots per node. We call this the *block frame*. Let B be the number of time slots in a block frame, then $B = N \lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \rceil$.

Each block frame shall place the D bits of information pertaining to each node in sequence according to i (n_i's assigned identifier), in ascending order. Let D_i denote the D bits of node n_i, then a block frame contains the sequence $D_0, D_1, \ldots, D_{N-1}$ for each node.

C. Super Frames

Let B_i denote a block frame for node n_i. Considering all N nodes, we would have NB corresponding to all time slots used to disseminate ND bits of information among N nodes. The NB slots forms a *super frame*. Then, there are

$S = N^2 \left\lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \right\rceil$ time slots in a super frame.

Example 2: Consider Figure 3, where node 0 contains the information bits 11010100. Although the number of

Fig. 3. Node 0 transmits to 7 other nodes in multihop-hop manner.

time slots need not be restricted to the number of nodes, we fix the number of time slots to 8 in this example. Node 0 partitions the bits into two sets, 1101 and 0100. A sync pulse is used to align all node timing. Node 0 transmits a 1 in time slot 6 in the first frame (effectively transferring information bits 1101) and then in the second frame transmits a 0 in time slot 2 (effectively transferring information bits 0100).

Let $\kappa_{D,N} = \left\lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \right\rceil$. The gain in reduced number of transmission from a single node containing D bits of information is stated in the following Theorem.

Theorem 3: For a network of N nodes, where a node contains D bits of information to be disseminated to the other $N - 1$ nodes, the ratio of reduction in the number of bits transmitted using TSIID is $\frac{\kappa_{D,N}}{D}$.

Proof of Theorem 3: For a node intending to disseminate D bits of information, we have a total of $\frac{D}{\lfloor \log_2 N \rfloor + 1}$ groups, where each groups contains $\lfloor \log_2 N \rfloor + 1$ bits. Since $\frac{D}{\lfloor \log_2 N \rfloor + 1}$ may not be an integer, in order to transmit the last partial group, we round up to the nearest integer. Dividing this by the total number of bits contained in the node completes the proof.

Figure 4 shows that the percentage of reduction in the total amount of improvement asymptotically approaches a constant for a set of nodes. By removing the ceiling and floor functions, we can lower bound the percentage reduction by $\frac{1}{\log_2 N + 1}$ which is fixed for any size D.

IV. NETWORK TOPOLOGIES AND INFORMATION DISSEMINATION

Our TSIID method works for networks, independent of the network topology. However, it can be complicated to obtain a theoretical bound for ϖ_G, where G is of arbitrary topology. Therefore, we consider three regular topologies for wireless networks: the line, star and grid. For each topology, we examine the number of transmissions required

Fig. 4. Reduction in number of required transmissions per D bits for various number of nodes in the network.

to propagate D bits of information using TSIID. We assume that all nodes know the topology of the network.

For nodes n_i and n_j with locations l_i and l_j respectively, they may communicate (or are connected) if $\|l_i - l_j\| \leq d$.

A. The line

For a line topology, as depicted in Figure 5, the network is composed of N nodes, spaced at d units apart in a straight line. Assuming each node contains D bits of information to

Fig. 5. A line of n nodes

be disseminated, we obtain the following theorem.

Theorem 4: For N nodes spaced an RF distance d apart, where each node contains D bits of information, the total number of single bit transmissions required to disseminate all ND bits of information using TSIID is $\kappa_{D,N} \varpi_{line}$, where $\varpi_{line} = (N^2 - 2N + 2)$.

Proof of Theorem 4: For the outer-most nodes, we have $N - 1$ transmissions to propagate a bit of information to all other nodes. For all other inner nodes, we have $N - 2$ transmission to propagate a bit of information.

B. The Star

We assume there exists a central node n_0. Nodes are placed at a distance kd units from the central node, such that k is the tier and that a node placed on tier $k + 1$ is exactly d units from the node placed on tier k, as shown in Figure 6.

Lemma 5: Suppose we have k equally spaced nodes per tier, where $k < 6$ (to disable communication among nodes in the same tier), and m tiers. Then the total number of transmissions required to disseminate D bits of information from the $N = km + 1$ nodes is $\kappa_{D,N} \varpi_{star}$, where $\varpi_{star} = (N^2 + (m - 1)N - 2k - k^2 + m + 1)$.

Sketch Proof of Lemma 5: For any broadcast, nodes must transmit through n_0 to propagate a bit to all other nodes.
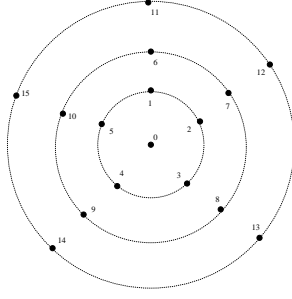
Fig. 6. A multi-tier star configuration, 3 tiers 5 nodes per tier

Broadcast from n_0 requires $1 + k(m - 1)$ transmissions. From a node other than n_0 and not on the mth tier, we have $(m - 1) + [1 + (k - 1)(m - 1)] = km - k + 1$. A node on the mth tier requires $km - k + 2$ transmissions. Note that $N = km + 1$, thus $\varpi_{star} = 1 + k(m-1) + m(km-k+2) + (m-1)k(km-k+1) = N^2 + (m-1)N - 2k - k^2 + m + 1$.

C. The Grid

In [1], the number of transmissions used per node in multicast gossip is upper bounded by $T = \frac{N + 3\sqrt{N} - 2}{2}$. This implies $T_N = N \times \frac{N + 3\sqrt{N} - 2}{2}$ for multicast gossip for all N nodes. Here, we present an algorithm that improves the bound stated above, without using TSIID.

The $k \times k$ grid contains $N = k^2$ nodes (see Figure 7(a)). We normalize the transmission radius $d = 1$, so that adjacent nodes are placed at distance d apart. The ith node is designated by n_i. Node numbering is by row major with n_0 at the lower left corner.

(a) $k \times k$ grid node identification

(b) A_0 graph for a $k \times k$ grid
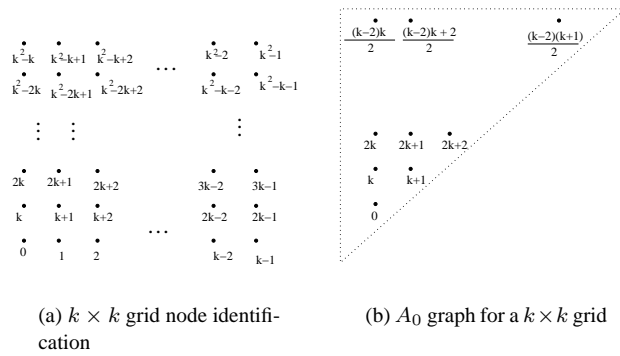
We define $A_0 \subset A$ as the subset of nodes, such that for any two nodes in A_0, there is a unique route. In other words, A_0 is the smallest subgraph that characterizes all possible shapes of paths in the network. Graphically, for k even in a $k \times k$ grid, Figure 7(b) depicts a typical A_0 subset.

Lemma 6: For a $k \times k$ grid, there exists a set of nodes A_0 which induces a subgraph containing the minimum number of unique paths for communication. The total number of nodes in the set, $S(A_0)$ is $|S(A_0)| = (\lfloor \frac{\sqrt{N}+1}{2} \rfloor +$

$1) \frac{\lfloor \frac{\sqrt{N}+1}{2} \rfloor}{2}$.

Proof of Lemma 6: Note that a $k \times k$ grid can be partitioned into 4 sets along the diagonals from the corners, where nodes touching the diagonal lines could be in either set. Add a vertical and a horizontal line bisecting the columns and rows will split the nodes into 8 sets. Suppose we select the upper triangle of the lower quadrant (square) to be $\hat{A}$. There are $\lfloor \frac{k+1}{2} \rfloor$ rows and columns in the set; so that the size of $\hat{A}$ is $S(\hat{A}) = \frac{1}{2}(\lfloor \frac{k+1}{2} \rfloor + 1)^2$. Recognize that all other nodes in the grid are transpositions and/or reflections along the rows and columns, and that set A_0 is set $\hat{A}$, and since $N = k^2$, we are done.

We now state some useful notation that we use in the proof below. Let the indicator function $I(x)$ be $I(x) = 1$ for $x > 0$ and $I(x) = 0$ for all other x. Let

$$G(k) = k + \lfloor \frac{k}{3} \rfloor (k - 2) + I(\text{mod}_3(k + 1) - 1)$$
$$+ \lfloor \frac{k - 2}{3} \rfloor (\text{mod}_3(k) + 1) I(\text{mod}_3(k)) . \quad (1)$$

D. Modulo 3 Routing Algorithm (M3RA) on a grid

We present a broadcast algorithm which improved the local communication of [1].

1. Choose a node $\hat{n} \in A_0$.

2. If the node is **not** in column 2, broadcast and choose a neighbor who is closer to column 2. Repeat this until the selected node is in column 2.

3. The node is in column 2, broadcast. Select the nodes above and below to repeat this process until all nodes in column 2 have received and transmitted a broadcast.

4. Select and broadcast for nodes in row $2 + 3i$, column 3, where $i = 0, \cdots$, such that $2 + 3i < k$. For each row $2 + 3i$ selected, increment the column by 1 and broadcast. Repeat this until all columns greater than 3 for the specified row have transmitted to neighbors.

5. If k is not a multiple of 3, then for the columns $5 + 3j < k$, choose row $\hat{i} = \max\{2 + 3i\}$ for all i valid from step 4 and broadcast from this set of nodes. For each of these selected nodes, after broadcasting, increment the row to select the next node. The process is repeated until all rows above $\hat{i}$ for the selected nodes have transmitted to neighbors.

A potential extraneous node may exist. For $k = 4 + 3m$, where $m = 0, 1, 2, \ldots$, select the node in row k column $k - 1$ and broadcast to the final neighbor in upper right corner.

Theorem 7: For $k > 3$, the total number of transmissions for multicast gossip in a $k \times k$ grid in an error free environment is bounded by

$$\varpi_{grid} = \begin{cases} N \cdot G(\sqrt{N}) + \frac{N^{\frac{3}{2}} - 9N + 50\sqrt{N} - 48}{6} & , \text{even } k \\ N \cdot G(\sqrt{N}) + \frac{N^{\frac{3}{2}} - 9N + 20\sqrt{N} - 78}{6} & , \text{odd } k \end{cases}$$

where $k = \sqrt{N}$ and each node initially contains a single bit

of information. Similarly, for each node containing D bits of information, we have $D \varpi_{grid}$.

Sketch Proof of Theorem 7: Without loss of generality it is sufficient to choose a node $\hat{n}$ from A_0. We claim that for node $\hat{n}$ in row r and column c,

$$T_{0,c,k} = |c-2| + \lfloor \frac{k-2}{3} \rfloor (\mathsf{mod}_3(k)+1)I(\mathsf{mod}_3(k))$$
$$+k + \lfloor \frac{k}{3} \rfloor (k-2) + I(\mathsf{mod}_3(k+1)-1) \,.(2)$$

Recognize that it requires exactly $|c-2|$ transmissions to reach the second column for any node in A_0, which is the first term on the right hand side of (2). For a $k \times k$ grid, it requires k transmissions to broadcast from each node in a single column. Moreover, all adjacent nodes to the specified column receive the broadcast. Thus, for column 2 we have the third term on the right hand side of (2), where k transmissions are required such that the first 3 columns of the grid have received a transmission not including the initial node.

Since all nodes in the first three columns have received a transmission, we can choose any node in column 3 as a broadcast point. The algorithm selects a node in the second row and spaces the next broadcast in column 3 spaced 3 rows above which is equivalent to grouping complete sets of 3 rows. Since we need complete sets of 3 rows for each of the transmissions from the third column, we have a total of $\lfloor \frac{k}{3} \rfloor$ sets of 3 rows. Recognize that for each set of 3 rows in column 3, transmissions required to cover all nodes in a single set of three rows is 1 to begin the process and $k-3$ to complete the column. Thus, we have for step 4, a total of $\lfloor \frac{k}{3} \rfloor (k-2)$ transmissions, which is the fourth term on the right hand side of (2).

Recognize that for step 5, the transmission contribution is highly dependent on k. Specifically, for $\mathsf{mod}_3(k)$ equal to zero, the broadcast is finished in step 4.

Suppose $\mathsf{mod}_3(k)$ is nonzero. Nodes with columns greater or equal to 5 in the highest valid transmit row are chosen in groups of 3 columns starting from column 2 but not including column 2. Thus, we have $\lfloor \frac{k-2}{3} \rfloor$ sets of three complete columns. For each column, we require $\mathsf{mod}_3(k)+1$ transmissions, resulting in a total of $\lfloor \frac{k-2}{3} \rfloor (\mathsf{mod}_3(k)+1)$ transmissions for all complete columns, which is essentially the second term on the right hand side of (2).

Since, incomplete columns exists for grid size $k = 4 + 3i$, for $i = 0, 1, \ldots$, an additional transmission covers the remaining node in the top right corner of the grid. Thus, we have $I(\mathsf{mod}_3(k+1)-1)$, which is the last term on the right hand side of (2). Rewriting (2) using (1), we have $T_{0,c,k} = |c-2| + G(k)$.

For even k, recognize that for the entire grid of all k^2 nodes, the equivalent are repeated nodes from A_0 with varying multiplying factors. Specifically, for each node in A_0 such that the row is equal to the column, for the entire grid,

each node is repeated 4 times. Similarly, all other nodes in A_0 are repeated 8 times in the entire grid. Thus, we have

$$\varpi_{grid} = \sum_{c=1}^{k/2} \left\{ 4T_{0,c,k} + 8 \sum_{r=c+1}^{k/2} T_{0,c,k} \right\}$$
$$= k^2 G(k) + \frac{k^3 - 9k^2 + 50k - 48}{6} \,.$$

The bound for odd k can be derived similarly. Let

$$Z_N = N[\sqrt{N} + \lfloor \frac{\sqrt{N}}{3} \rfloor (\sqrt{N} - 2)] \,,$$

$$L_N = \begin{cases} Z_N + \frac{N^{\frac{3}{2}} - 9N + 50\sqrt{N} - 48}{6} & , k \text{ even} \\ Z_N + \frac{N^{\frac{3}{2}} - 9N + 20\sqrt{N} - 78}{6} & k \text{ odd} \end{cases} \,,$$

and

$$U_N = Z_N + 3N \lfloor \frac{\sqrt{N} - 2}{3} \rfloor + N + L_N \,,$$

where $N = k^2$ and k is an integer.

Corollary 8: For N nodes in a $k \times k$ grid where $N = k^2$, the total number of transmissions required to broadcast in a multicast gossip manner is upper and lower bounded by U_N and L_N respectively.

Proof of Corollary 8: Since

$$G(k) \quad \geq \quad k + \lfloor \frac{k}{3} \rfloor (k-2) \,,$$

for k even, we have

$$T(k) = k^2 G(k) + \frac{k^3 - 9k^2 + 50k - 48}{6}$$
$$\geq k^2(k + \lfloor \frac{k}{3} \rfloor (k-2)) + \frac{k^3 - 9k^2 + 50k - 48}{6} \,.$$

Since

$$G(k) \leq k + \lfloor \frac{k}{3} \rfloor (k-2) + 3 * \lfloor \frac{k-2}{3} \rfloor$$
$$+ I(\mathsf{mod}_3(k+1) - 1)$$
$$\leq k + \lfloor \frac{k}{3} \rfloor (k-2) + 3 * \lfloor \frac{k-2}{3} \rfloor + 1 \,,$$

for k even, we have

$$T(k) = k^2 G(k) + \frac{k^3 - 9k^2 + 50k - 48}{6}$$
$$\leq k^2(k + \lfloor \frac{k}{3} \rfloor (k-2) + 3 * \lfloor \frac{k-2}{3} \rfloor + 1)$$
$$+ \frac{k^3 - 9k^2 + 50k - 48}{6} \,.$$

Bounds for odd k are derived similarly.

Theorem 9: [M3RA-TSIID] For a $k \times k$ grid with each node containing D bits of information, using M3RA, the total number of transmissions for multicast gossip using TSIID, is $T_{TSIID}(D, N) = \kappa_{D,N} \varpi_{grid}$.

Proof of Theorem 9: Since $\lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \rceil$ corresponds to the total number of bits transmitted from a single node, then using Theorem 7, where ϖ_{grid}/N is equivalent to the average number of transmissions from a single node, we have the effective number of transmissions as

$$
\begin{aligned}
T_{TSIID}(D, N) &= N \lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \rceil \varpi_{grid}/N \\
&= \lceil \frac{D}{\lfloor \log_2 N \rfloor + 1} \rceil \varpi_{grid} .
\end{aligned}
$$

Recognize that

$$
\lceil \frac{1}{\lfloor \log_2 N \rfloor + 1} \rceil > \frac{1}{\lfloor \log_2 N \rfloor + 1}
$$

and so we can consider as a lower bound, for the $D = 1$ case, we have

$$
T_{TSIID}(1, N) > \frac{1}{\lfloor \log_2 N \rfloor + 1} \varpi_{grid} .
$$

Recognize that if each node contained only 1 bit to transmit (i.e. Each node has $D = 1$), then there is no gain in applying TSIID. However, there is a clear gain in using TSIID as D increases.

E. Grid with Lossy Links

We now consider a $k \times k$ grid with probability p of symbol error on links. In each time slot, the probability of a received bit being erroneous is p, and the probability of a received bit being error-free is $1 - p$. Since each bit transmission could result in up to $\log_2 N + 1$ bit errors (equivalent to a symbol error) at any receiving node, there is a possibility of a symbol error at up to 3 possible receiving nodes in a grid. Thus, we have the probability of at least one of three nodes indicating that symbol error has occurred in a slot of time is $3p - 3p^2 + p^3$. Thus, we have an upper bound on the probability of an error occurring in a slot of time for each hop in a broadcast.

Let T be the random variable representing the total number of transmission required to multicast-cast gossip among N nodes. Let $K(p) = \frac{(1-p)}{(1-3p+3p^2-p^3)^2}$.

Theorem 10: Using M3RA, the average number of transmissions required to multicast gossip D bits over a channel with symbol error probability p is upper and lower bounded by $K(p)DU_N$ and $K(p)DL_N$ respectively.

For M3RA-TSIID, the upper and lower bound on the total number of transmissions is $K(p)\kappa_{D,N}U_N$ and $K(p)\kappa_{D,N}L_N$ respectively.

Proof Theorem 10: Without loss of generality, we consider only the upper bound. Using the upper bound on the number of independent hops from Corollary 8 and $\hat{p} = 3p - 3p^2 + p^3$, to propagate a symbol of information among $N - 1$ nodes, we have

$$
\begin{aligned}
E[T] &= \sum_{i=1}^{U_N} \sum_{t=1}^{\infty} tPr\{ \text{ first error free slot } t \} \\
&\leq \sum_{i=1}^{U_N} \sum_{t=1}^{\infty} t\hat{p}^{t-1}(1-p) \\
&= (1-p)U_N \frac{1}{(1-\hat{p})^2} .
\end{aligned}
$$

Replacing $3p - 3p^2 + p^3$ for $\hat{p}$, and since the above result is for a single symbol from each node, multiply this by D for D symbols completes the proof.

Note that in Theorem 10, $K(p)$ may be viewed as the multiplying factor due to errors.

F. Performance Comparisons

We now report the total number of transmissions for the regular topologies discussed above. In Figure 7, we see the effect of the TSIID used for a line, a multi-tier star, and a grid. For the star topology, we use the minimum number of tiers while maintaining the identical number of nodes per tier and insuring less than 6 equally spaced nodes residing on each tier. We see that the multi-tier star topology
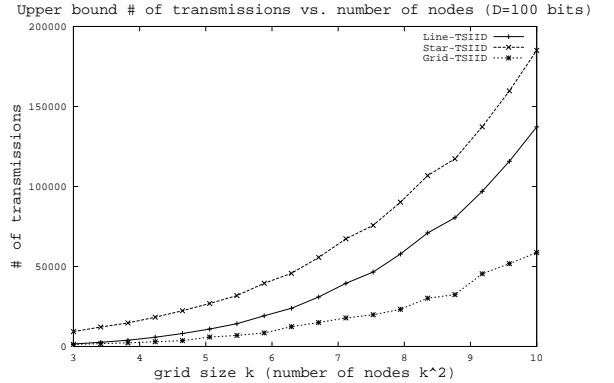

Fig. 7. Number of transmissions comparison between a line, star, and grid topology

is at a disadvantage because all routes must pass through the central node; the grid topology with the M3RA routing algorithm has the best performance.

A simulation was performed for A_0 transmission ordering to multicast gossip to all other nodes, considering error-free transmissions. Figure 8 shows the total number of transmissions increasing on the order of $O(k^2)$ for the grid. For verification and comparison, we also plotted the theoretical upper and lower bounds (Corollary 8) as well as the original bound obtained in [1]. The plot confirmed that our upper bound has improved the previous bound stated in [1].

The original bound (4-3) of [1] is not as tight as the M3RA upper bound. Although this is an improvement on
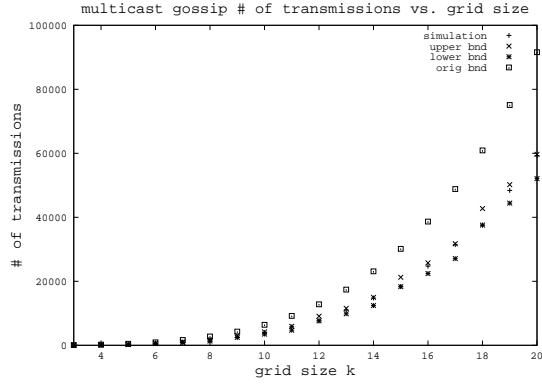
Fig. 8. Number of total transmissions to broadcast 1 bit of information from each node to all other nodes in a $k \times k$ grid without TSIID
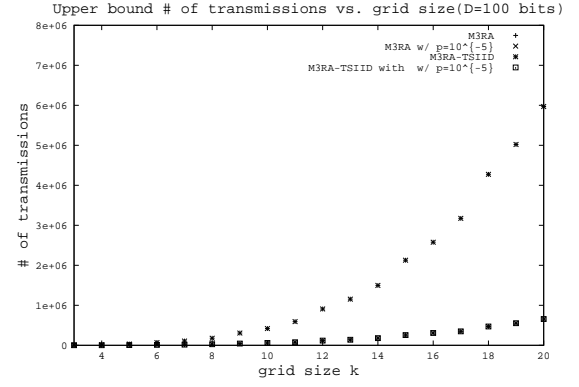

Fig. 10. Upper bound number of transmissions for M3RA versus M3RA with TSIID

the total number of transmissions required to propagate a bit of information, further studies are required to tighten the bound using simpler routing algorithms.

For a link error event, we do not consider the number of acknowledgments or overhead required in recognizing an error event. It can be shown that the multiplying factor is marginal for error rates of $p = 10^{-5}$ and below suggesting that the added effect due to a lossy link is minimal toward the total number of transmissions required to multicast gossip.
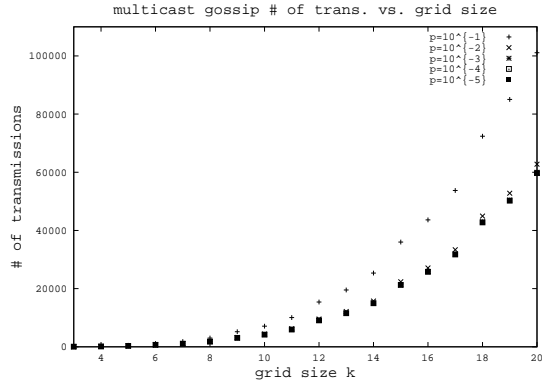

Fig. 9. Upper bound number of transmissions with various number of bit errors

In Figure 9, negative contributions due to various probabilities of symbol ($\log_2 N + 1$ bits) errors are factored in (as in Theorem 10), validating the claim that there is a marginal negative contribution due to link error events. In Figure 10, for both error-free and reasonable probability of bit errors ($p < 10^{-5}$), the TSIID upper bound of the total number of transmissions (for $D = 100$ bits transmitted) provides significant gain over a scheme not utilizing TSIID. Note that the energy required to compensate for the symbol error must be increased synonymous to the work in [1]. However, schemes such as gray coding the bit sequences should reduce the effect.

V. DISCUSSION

In this paper, we leveraged bi-orthogonal signaling on the hard decision data for information dissemination by presenting a time slot scheme that reduces the number of transmissions from a given node. TSIID could in fact operate in conjunction with a version of the optimal receiver of Figure 1 although the overall performance as suggested in [1] is upper bounded by leveraging all orthogonality on the physical layer implying a trade-off in receiver complexity versus bit error performance for a given amount of total transmit energy.

Finally, we presented an algorithm that tightens the bound on the information dissemination for nodes in a grid. We recognized that the overall negative effect on the total number of transmissions in the network of nodes is marginal for low symbol error rates.

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